Social welfare distribution in a multiperiod auction on a pool-based electricity market

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ABSTRACT. A new scheme is proposed for determining the social welfare distribution for the multi-period auctions in the context of solving the unit commitment problems. The approach is as close as possible to a competitive electricity market clearing, i.e. the generating units are paid uniform hourly selling energy prices and consumers are paid uniform hourly buying prices. Differences in buying and selling prices enable to cover the costs of necessary compensations paid to unfairly priced participants. The pricing model is in the form of a mixed linear programming model that minimizes the compensation costs. The model allows every thermal unit and every consumer to obtain individual maximum profits.

Keywords. Electricity market, energy pricing, thermal units, dynamic and integer programming, unit commitment.

1. INTRODUCTION

The pool-type auctions are an important group of possible spot market designs that may be well suited coordination mechanisms for the short-term trading of energy on electricity markets. The basic goal of the pool can be to the maximize the net social welfare through the production and consumption of electricity.

The pool scheduling problem can be stated in a form of the unit commitment optimization model for a day-ahead multi-period auction, that balances production offers with constant (or elastic) demand. The model can be formulated as a mixed integer linear (MILP) or nonlinear (MINP) programming problem that takes into account security constraints, the unit capacity constraints, and production characteristics of thermal units, including start-up and shut-down ramp constraints, minimum up and minimum down commitment constraints, etc. The spot market rules must assure feasibility of the system operation together with efficient energy production and maximization of the social welfare objective function, see Arroyo and Conejo (2002), Baldick et al. (2005).
The appropriate rules for market pricing of energy and generation services are the key spot market design issues. It is now widely recognized that fair market clearing and determination of welfare distribution cannot be performed upon the uniform marginal prices scheme due to the indivisibilities and non-convexities existing in the pool-based market balancing models. No uniform commodity marginal prices that support an equilibrium exist, for which the profit-driven independent generators would self-schedule to meet the demand. Therefore, marginal pricing of the energy only is not sufficient to support equilibrium allocations in a decentralized auction-based market and to assure efficient self-committed schedules, (Galiana et al., 2003; Garcia-Bertrand et al., 2005; Doorman and Nygreen, 2003; Madrigal and Quintana, 2001).

A new promising approach for market clearing pricing and fair social welfare distribution developed in Toczyłowski (2002) is based on differentiating the sell and buy competitive prices of some commodities and services. Based on that approach, in this paper we have designed a new model for fair social welfare distribution in the multi-period auctions in the context of solving the unit commitment problems. The model can be considered as a separate pricing step of the market clearing procedure, after the social welfare has been maximized by solving a unit commitment problem. The constraints that are considered in the model may include the minimum generation level, the minimum start-up times and costs. Our approach is as close as possible to the competitive electricity market clearing, i.e. the generating units are paid uniform hourly selling energy prices and consumers are paid uniform hourly buying prices. The model gives some incentives to generators to bid fairly and allows every generator (thermal unit) and every buyer to obtain individual maximum profits.

In section 2 we discuss various approaches for social welfare distribution in the multi-period auctions. Detailed formulation of the generic pricing model is reported in section 3. It is in the form of a mixed linear programming model that minimizes the sum of the compensation costs. Furthermore, the multi-period commitment compensations are introduced in section 4. In section 5 we illustrate the social welfare distribution on example that is based on Arroyo and Conejo (2002). We show how the market clearing prices are computed with the newly proposed model and analyze their fairness properties.

2. SOCIAL WELFARE DISTRIBUTION PROBLEM

In the pool auction markets the sellers (and buyers) submit sale bids (and purchase bids) to a central dispatch office, or market Operator. The Operator collects all offers and solves a unit commitment program that computes a minimum-cost optimal scheduling strategy to balance production and consumption every time. The main goal of the Operator may be to maximize the social welfare, which is defined as the sum of the consumer surplus and the
producer surplus (Arroyo and Conejo, 2002).

That solution procedure is reminiscent of the economic dispatch and unit commitment programs that were used in the pre-competition times. However, a fundamental difference is that sellers are self-interested decision-makers driven by the need to achieve high individual profits. There are conflicts between the centralized maximum welfare goal and the profit-driven goals of the independent sellers.

![Figure 1: Intersection of supply and demand side may not be an equilibrium on a pool-based multiperiod auction](image)

The centrally imposed schedule may require some costly generators to operate, while scheduling other competitive units off, even though these would have make profit by turning on — see Figure 1. The rationality assumption of self-interested sellers under competition is that no seller can willingly accept a loss of profit, if such loss can be avoided by turning some units off-line. To cope with market requirements, some market designers have established various rules that can be imposed on the participants with some degree of arbitrariness and equity. Some market rules can also help the participants to satisfy their requirements. For example, as in the Spanish market, the sellers can be allowed to specify a minimum income requirement in their hourly portfolio bids.

Even though the unit commitment models allow us to maximize the social welfare under generator operating constraints, the fair market-clearing prices for each hour cannot be explicitly obtained from such models. This fact, known as the lack of the competitive equilibrium on a pool-based market was observed by many authors, see Doorman and Nygreen (2003); Galiana et al. (2003); Garcia-Bertrand et al. (2005); O’Neill et al. (2005). Setting fair energy prices in the multi-step auction is a very challenging problem. There are a number of various approaches for spot market clearing pricing in the multi-period auctions.

In the first approach the hourly spot energy market segments are cleared independently, and the hourly prices of single-period equilibria are found at the intersection of the aggregated simple buyers and seller bids. Such a market design requires the individual generators to ”internalize” nonlinear start-up and production costs through offers and neglects the time-coupling constraints. That approach may imply the cross-agent subsidies and poses important risk to generators, since only the generating units are responsible for satisfying
their dynamic coupling constraints.

The second approach (Garcia-Bertrand et al., 2005) is based on solving a multi-period equilibrium problem with non-convexities, where the optimization model can be in the form of a large-scale unit commitment problem with price-related nonlinear constraints (for instance, by enforcing minimum profits for power producers). Such problems can be solved by using the Benders decomposition, with the master problem that sets the on/off binary variables describing the status of the generations units, and the market equilibrium subproblems of the generators, consumers and the system operator (for fixed status variables). The multi-period equilibrium is defined by the optimality conditions of the optimization problem with the social welfare profit maximization objective. However, the "market" prices that support a competitive equilibrium are the shadow prices of many individual constraints and therefore they can be hardly considered as fair market prices of some "commodities". In particular approach (O’Neill et al., 2005), the integral activity conditions of the individual agents can be priced by inserting linear equality constraints (cuts) that force the integer variables to assume their optimal values in the resulting linear programs. However, the dual prices that are calculated to support a competitive equilibrium are different for different generation units, and this feature is hardly justified by the fairness expectations.

In (Madrigal and Quintana, 2001) it is shown that the production cost not recovered by Lagrange multipliers can be used to define the nonuniform prices. The prices are computed as increments or decrements of marginal prices and they allow to pass the costs in equal proportion, to the suppliers and the demand.

Another family of approaches is based on designing a way of providing some compensations paid to the market participants, that are separate from the market-clearing price payments (Toczyłowski, 2002; Galiana et al., 2003). The general idea is to share more or less evenly some social costs (compensation costs or profit-optimality loss) between the consumers and the generators, in order to treat all market participants without discrimination and provide the market Operator with more degrees of freedom to properly control admissible strategies of the market participants. The sharing of this costs is justified by the incentives it should give to the market participants to bid fairly.

One such compensation approach is based on providing uplifts (and generalized uplifts) paid to or from generators only, separate from market-clearing price payments. Representative, optimization-based methodologies are proposed in Galiana et al. (2003); Bouffard and Galiana (2005). The incongruence between the minimum-cost centralized schedule and the self-schedule is reduced through the uplifts. The generator that is scheduled on, but incurs a financial loss under minimum cost schedule, is assigned an uplift, i.e., an amount that compensates the generator by just enough to break even. Self-scheduling is then done by
maximizing the profit together with uplift. The resulting uplift costs are charged entirely
to the consumers by increasing the consumer tariffs. Under such rules, the consumers are
the ones treated inequitably, as the generators profits are achieved through a subsidy solely
obtained at the expense of the loads. The generalized uplift approach spreads the loss of
profit equitably among both consumers and producers while ensuring market equilibrium.
The sharing of the profit sub-optimality costs is justified by the incentives it should give to
the generators not to try to artificially cause profit sub-optimality by their offering strategies.
The concept of the generalized uplifts applies to all generators, either scheduled on or off.
In order to specify the uplifts, the minimum generalized uplift optimization problem must
be solved. However, as it is a complex non-convex and highly combinatorial optimization
problem, it can be hard to solve (Bouffard and Galiana, 2005).

Another compensation approach for setting fair energy prices is based on differentiating the
sell and buy competitive market prices $\pi_S, \pi_B$ to cover the costs of necessary compensations
$R$ paid to unfairly priced participants (Toczyłowski, 2002). The number of commodities and
services cleared in this way can be care fully limited in the model, to assure liquidity of the
market. The total compensation cost is calculated separately to the profit-optimality loss
and these two objective functions are the objectives of a multi-objective optimization market
clearing problem. In the model, the social welfare is fairly redistributed among agents by
minimizing both the overall compensation cost $R$ and the profit-optimality loss. The sharing
of the compensation costs allows the market Operator to treat all market participants without
discrimination and is justified by the incentives it should give to the market participants to
bid fairly. The approach is as close as possible to a competitive electricity market clearing,
i.e. the generating units are paid uniform hourly selling energy prices and consumers are
paid uniform hourly buying prices, with the aim to keep the differences between the sell and
buy competitive prices as small as possible.

One Pareto-optimal solution of the problem can be calculated by assuming zero profit-
optimality loss, or equivalently, finding the optimal solution of the centralized market clearing
problem that maximizes the social welfare. Then, in order to obtain the best sell and
buy competitive prices $\pi_S, \pi_B$, the compensation cost $R$ is minimized by solving a simple
compensation and price determination (linear) programming problem. In this problem the
components of the compensation cost $R$ are calculated as functions of the surplus prices of the
rejected competitive offers and functions of the deficit prices of the accepted uncompetitive
offers. The way of calculating the compensation cost $R$ is described in more details in
section 3.

In this paper we present a new model for minimizing the compensation cost $R$ and fair
social welfare distribution for the multi-period auction in the context of solving the unit
commitment problem. For simplicity of the presentation, we shall use the particular unit
commitment optimization model for a day-ahead multiperiod auction comprising thermal units and elastic demand presented by Arroyo and Conejo (2002). This model is in the MILP form and it takes into account the unit capacity constraints, start-up and shut-down ramp constraints, minimum up and minimum down commitment constraints, etc.

3. COMPENSATION AND PRICE DETERMINATION PROBLEM

Let $M$ be a set of consumers, $L$ set of generating units and $H$ set of hours of the market horizon. Each participant submits bids consisting of $I$ blocks defining energy-price pair in every hour. We state $c_{mhi}$ as a price of $d_{mhi}^{\text{max}}$ maximum amount of energy offered to buy in the $i$th block of $m$th consumer bid in hour $h$ (Figure 2(a)). Analogously, $s_{lhi}$ is a price of $p_{lhi}^{\text{max}}$ maximum amount of energy in the $i$th block of $l$th unit bid in hour $h$ (Figure 2(b)). The first offered energy block $p_{lhi}^{\text{max}}$ states the minimum capacity of $l$th unit. Thus, to avoid manipulations, we propose constant sell bids, although this is not required in the model. Furthermore, each $l$th unit is allowed to specify the minimum up time $T_l$ and start up price $S_l$.

![Figure 2: Structure of hourly offers in considered auction: $i \in I$ blocks of energy-price pairs in every hour $h$](image)

We assume that in the first step of the market clearing procedure the market Operator maximizes social welfare by solving the unit commitment optimization model of the same type as in (Arroyo and Conejo, 2002). The solution provides the accepted quantities of the energy buy and sell bids, that is every energy $d_{mhi}$ consumed from $i$th block of $m$th consumer’s bid in hour $h$, and every energy $p_{lhi}$ produced in $i$th block of $l$th unit’s bid in hour $h$. The accepted sell bids provide also the state of the commitment variables $v_{lh} = 1$ if $l$th unit is on in hour $h$ and $r_{lh} = 1$ if it is started-up in hour $h$. All of these preserve technical constraints.

Our price determination model may now be applied as a separate pricing step of the market clearing procedure, after the social welfare has been maximized. It allows the market Operator to fairly redistribute the social welfare among agents, by computing sell and buy hourly market prices that minimize the costs of necessary compensations. As it was mentioned
before, those compensations should be paid to avoid individual profit-optimality losses, that may occur due to nonconvexities existing in the market.

Minimization of the compensation costs $R$ leads to the minimization of differences between buying and selling prices (Toczyłowski, 2002)

**Objective:**

$$
\min_{\pi^B, \pi^S} R = \sum_{h \in H} D_h (\pi^B_h - \pi^S_h)
$$

where $D_h$ is the total power consumed and produced in hour $h$, $D_h = \sum_{m \in M} \sum_{i \in I} d_{mhi}$, $D_h = \sum_{l \in L} \sum_{i \in I} p_{lhi}$. Variables $\pi^B_h$ and $\pi^S_h$ are respectively buying and selling prices in hour $h$, preserving constraint $\pi^B_h \geq \pi^S_h$, $\forall h \in H$. It is interesting to observe that in the case of clearing a linear market model (without non-convexities), the model would produce the uniform market marginal prices, the same for buyers and sellers, i.e., $\pi^B_h = \pi^S_h$. In fact the differences in buying and selling prices will tend to zero as the number of bids in the auction increases. The larger the number of bidders in the auction, the smoother the supply function and the better the chances for an equilibrium to exist (Madrigal and Quintana, 2001).

Let us introduce surplus prices $\lambda^+_{mhi}$, $\lambda^+_{ithi}$ defined for $ith$ competitive block price of $mth$ consumer and $lth$ unit in hour $h$. The value of $\lambda^+_{mhi}$ is the value of the surplus benefit $mth$ consumer gains when the market price $\pi^B_h$ in hour $h$ is less than its $ith$ bidding price $e_{lhi}$: $\lambda^+_{mhi} = \max\{e_{mhi} - \pi^B_h, 0\}$. Similarly $\lambda^+_{ithi}$ is the value of surplus benefit $lth$ unit gains when the market price $\pi^S_h$ in hour $h$ is greater than its $ith$ bidding price $s_{lhi}$: $\lambda^+_{lhi} = \max\{\pi^S_h - s_{lhi}, 0\}$.

Analogously, we consider deficit prices $\lambda^-_{mhi}$, $\lambda^-_{lhi}$ defined for $ith$ uncompetitive block price of $mth$ consumer and $lth$ unit in hour $h$. The value of $\lambda^-_{mhi}$ is the value of the deficit that $mth$ consumer is loosing when the market price $\pi^B_h$ in hour $h$ is greater than its $ith$ bidding price $e_{lhi}$ while the offer is accepted: $\lambda^-_{mhi} = \max\{\pi^B_h - e_{mhi}, 0\}$. In the same way $\lambda^-_{lhi}$ is the value of the deficit $lth$ unit looses when the market price $\pi^S_h$ in hour $h$ is less than its $ith$ bidding price $s_{lhi}$ while the offer is accepted: $\lambda^-_{lhi} = \max\{s_{lhi} - \pi^S_h, 0\}$.

Surplus and deficit prices may be modeled in function of the market prices as:

$$
\lambda^+_{mhi} - \lambda^-_{mhi} = e_{mhi} - \pi^B_h, \ m \in M, h \in H, i \in I
$$

$$
\lambda^+_{lhi} - \lambda^-_{lhi} = \pi^S_h - s_{lhi}, \ l \in L, h \in H, i \in I
$$

Then, if any $mth$ buyer is forced to buy energy in blocks where bidding price is less then market price in hour $h$, he gets a compensation $R_{B,mh}$ equal to deficit he would loose:

$$
R_{B,mh} = \sum_{i \in I} \lambda^-_{mhi} d_{mhi}, \ m \in M, h \in H
$$

The same type of compensation is defined for any $lth$ unit forced to produce energy when offered price is greater then market price in hour $h$. There is only one exception, namely
when unit is producing at minimum cost. Due to multiperiod constraints and start up costs it may be profitable for unit to loose in some hours. Thus, compensation $R_{S,lh}$ is equal only to deficit faced when selling energy in second or consecutive blocks:

$$R_{S,lh} = \sum_{i=2}^{\mid I \mid} \lambda_{\text{thi}} p_{\text{thi}}, \ l \in L, h \in H$$

The illustrations of determining compensation costs of loosing deficit for buyers and for sellers is given in Figure 3.

![Figure 3: Determination of hourly compensations $R_{h}$ for loosing deficit based on calculating deficit prices $\lambda^{-}$ and market prices $\pi$ for submitted offers and accepted quantities](image)

Next, if any $m$th buyer gets less or none energy in block with offered price greater than market price in hour $h$, then he should get compensation $R_{B,mh}^{o}$ equal to lost opportunity of gaining surplus:

$$R_{B,mh}^{o} = \sum_{i \in I} \lambda_{mhi}^{+}(d_{\text{max}}^{mhi} - d_{mhi}), \ m \in B, h \in H$$

Most complicating element is defining lost opportunity of gaining surplus for units, because of their multiperiod constraints. Obviously, we can define compensation of lost profit $R_{S,lh}^{o}$ for any working $l$th unit in hour $h$:

$$R_{S,lh}^{o} = v_{lh} \sum_{i \in I} \lambda_{\text{thi}}^{+}(p_{\text{max}}^{\text{thi}} - p_{\text{thi}}), \ l \in L, h \in H$$

The illustrations of determining compensation costs of loosing profit opportunities for buyers and for sellers is given in Figure 4.

Notice, that both type of compensations $R_{S,lh}$ and $R_{S,lh}^{o}$ are nonzero only for working units in hour $h$. For units that are off-line, we must determine additionally, if they have incentives to change their status by analyzing the whole market horizon, and not only a single hour. Furthermore, units that are on-line might have also incentives to change their status, even though they get or get not compensations $R_{S,lh}$ or $R_{S,lh}^{o}$. That is the reason why we have to define additional commitment compensation $R_{S,lh}^{c}$ which should be paid to $l$th unit that has an incentive to change its state in hour $h$. This happens when the optimum pool schedule
is different from the unit’s self-schedule under market prices (Garcia-Bertrand et al., 2005). Method of calculating $R_{S lh}^c$ will be explained in the next section.

Finally, to ensure that the balancing mechanism is financially neutral, all costs must be recovered by charges:

$$Z - K = 0$$  \hspace{1cm} (8)$$

where $K$ is the total market value of energy produced by generators, equal to the sum of costs of selling energy upon market selling prices and increased by the compensations paid to unfairly priced units:

$$K = \sum_{h \in H, l \in S} \left( \pi_S^h \left( \sum_{i \in I} p_{l,h,i} \right) + R_{S lh}^c + R_{S lh}^o + R_{S lh}^c \right)$$  \hspace{1cm} (9)$$

and $Z$ is the market value of the energy consumed by loads, equal to the charges from energy paid upon the market buying prices reduced by compensations returned to unfairly priced consumers:

$$Z = \sum_{h \in H, m \in M} \left( \pi_B^h \left( \sum_{i \in I} d_{m,h,i} \right) - R_{B mh}^c - R_{B mh}^o \right)$$  \hspace{1cm} (10)$$

Now, summarizing, if one would neglect the commitment compensations $R_{S lh}^c$ in our compensation and price determination problem, the market price clearing model would become a simple LP problem defined by constraints (2)–(10), with (1) as the minimized objective.

4. **MODELING COMMITMENT COMPENSATIONS**

Multiperiod constraints, including the minimum start-up time and cost that are taken into account when balancing the market, pose significant difficulties when determining unit’s profitability of working in particular hours — see Figure 5. We shall calculate compensations by comparing the actual profit made when realising operational schedule to the optimal self unit-commitment profit derived from the forward dynamic programming. The optimal self
Figure 5: Incongruence between the minimum-cost centralized schedule and the self-schedule requires to analyze the whole market horizon to determine the unit’s incentives to change the on/off status.

Commitment is evaluated with the reference to the units’ prices and constraints that appear in the offer.

For unit $l$ we consider two states: committed or uncommitted for every hour in market horizon. Unit in an uncommitted state makes zero profit, whereas in a committed state it makes optimal profit $P_{lh}$ of selling energy at market price in hour $h$. This profit may be negative when unit’s offered prices are greater than the market price — the unit would run at the minimum capacity $p_{l h}^{\text{max}}$ then. By using the surplus and deficit prices defined in section 3 we compute the maximum profit $P_{lh}$ when producing in hour $h$ as:

$$P_{lh} = \sum_{i \in I} \lambda_{l h i}^+ p_{l h i}^{\text{max}} - \lambda_{l h i}^- p_{l h i}^{\text{max}} l \in L, h \in H$$

Next, in each state we shall compute the optimal value functions $\omega_{lh}$ for each unit $l$ and hour $h$. These functions describe how is it profitable for the unit $l$ to be in a given state 0/1 during a particular hour $h$. The calculations are based on the forward dynamic programming approach, so the optimal value is computed for optimal policy leading the $l$th unit from the initial stage of the market horizon until hour $h$ — Figure 6.

Optimal value function $\omega_{lh}^0$ computed in an uncommitted state 0 of unit $l$ in hour $h$ is equal to the maximum of optimal value functions in a previous hour:

$$\omega_{lh}^0 = \max \left\{ \omega_{l,h-1}^0; \omega_{l,h-1}^1 \right\}$$
The $\omega_0^l$ is always nonnegative, as in the first hour it is equal to zero:

$$\omega_0^l, 1 = 0 \quad l \in L$$  \(12\)

To compute optimal value function $\omega_1^l, h$ in the committed state 1 of unit $l$ in hour $h$ we must compare the optimal value function in a committed state in the previous hour and the optimal value function in an uncommitted state in hour $h - T_l$, reduced by the value of the start-up cost. This is because the transition from uncommitted to committed state is possible only after $T_l$ hours with start-up cost $S_l$. The optimal value function $\omega_1^l, h$ is equal to the maximum of those two values plus the profit $P_l$ made in a commitment state:

$$\omega_1^l, h = P_l + \max \{\omega_0^l, h - T_l - S_l; \omega_1^l, h - 1\}$$

Profit $\omega_1^l$ in the first hour is equal to:

$$\omega_1^l, 1 = P_l - r_l S_l \quad l \in L$$  \(13\)

Computation of profits $\omega^1, \omega^0$ in MIP formulation model requires introducing the binary variables $u_1^l$ and $u_0^l$. They may be considered as the state decision variables. Particularly, $u_1^l$ is equal to 1 if reaching committed state for $l$th unit in hour $h$ is most profitable after being uncommitted in previous hours, and is equal to 0 if reaching committed state is most profitable after being committed also in previous hour. Analogously, $u_0^l$ is equal to 1 if reaching uncommitted state for $l$th unit in hour $h$ is most profitable after being also uncommitted in previous hour, and is equal to 0 if reaching uncommitted state is most profitable after being committed in previous hour. The binaries used with a large quantity $\Upsilon$ guarantee proper realization of the max{} function:

$$0 \leq \omega_1^l, h - 1 \leq \omega_0^l, h - 1 + (1 - u_1^l) \Upsilon \quad l \in L, h = 2 \ldots |H|$$  \(14\)

$$\omega_1^l, h - 1 \leq \omega_0^l, h - 1 + u_0^l \Upsilon \quad l \in L, h = 2 \ldots |H|$$  \(15\)

$$\omega_1^l, h - 1 + P_l \leq \omega_0^l, h - 1 + P_l + u_1^l \Upsilon \quad l \in L, h = 2 \ldots |H|$$  \(16\)

$$\omega_0^l, h - T_l - S_l \leq \omega_1^l, h - 1 \leq \omega_0^l, h - 1 - S_l + P_l + (1 - u_1^l) \Upsilon \quad l \in L, h = T_l + 1 \ldots |H|$$  \(17\)
Obviously $u_{lh}^1 = 0$ when start-up is impossible because of the minimum up time, that is, when $h = 2...T_l$, $\forall l \in L$.

The maximum of optimal value functions $\omega_{lh}^1$ and $\omega_{lh}^0$ determined in the last period gives the optimal self unit-commitment profit. Unit that obtains lower profit when realising operational schedule gets nonzero commitment compensations $R_{S,l}^c$:

\[
R_{S,l}^c \geq \omega_{lh}^1 - \sum_{h \in H} (P_{lh}v_{lh} - S_{lr_{lh}}) \quad l \in L
\]
\[
R_{S,l}^c \geq \omega_{lh}^0 - \sum_{h \in H} (P_{lh}v_{lh} - S_{lr_{lh}}) \quad l \in L
\]
\[
R_{S,l}^c \geq 0 \quad l \in L
\]

Notice, that $P_{lh}$ is a maximum possible profit that $l$th unit can make in hour $h$ (equation (11)). For a unit actually working in hour $h$ ($v_{lh} = 1$) this is equal to profit obtained under market price plus eventually the hourly compensation $R_{S,lh}^c$ or $R_{S,lh}$.

The main purpose of a commitment compensations $R_{S,l}^c$ is to allow the operational schedule to become alternative optimal self-schedule. After computing the value of $R_{S,l}^c$, we can distribute it to these hours, when unit has an incentive to depart from the operational schedule. For any hour, except the last, this happens when there leads no optimal policy from one to another consecutive state in operational schedule — Figure 7. Obviously, compensation should be paid also in the last period if the maximum profit is obtained in a state opposite to operational schedule.

![Figure 7: Assigning commitment compensations to periods when operational state is last on optimal policy path](image)

The values of commitment compensations assign to particular hours can be calculated based on surpluses of optimal value functions $\omega_{lh}^1$, $\omega_{lh}^0$ in the state opposite to operational and the...
operational state. Very simple LP model may be stated for given values of $\omega^v$:

$$\min \sum_{h \in H} R^c_{S, lh}$$  \hspace{1cm} (21)

$$R^c_{S, lh-1} \geq (1 - v_{lh})(1 - 2v_{lh-1})(\omega^1_{l, h-1} - \omega^0_{l, h-1}) \ l \in L, h = 2...|H|$$  \hspace{1cm} (22)

$$R^c_{S, lh-1} \geq v_{lh}(2v_{lh-1} - 1)(\omega^0_{l, h-Ti} - \omega^1_{l, h-1} - S_l) \ l \in L, h = Ti + 1...|H|$$  \hspace{1cm} (23)

$$R^c_{S, lH} \geq (1 - 2v_{lH})(\omega^1_{lH} - \omega^0_{lH}) \ l \in L$$  \hspace{1cm} (24)

$$R^c_{S, lh} \geq 0, l \in L, h \in H$$  \hspace{1cm} (25)

Although hourly values $R^c_{S, lh}$ could be calculated simultaneously when computing market prices, it is much more efficient to compute $R^c_{S, l}$ only. It can be shown that the values $R^c_{S, lh}$ defined in model (21)–(25) sum up exactly to $R^c_{S, l}$. The compensations $R^c_{S, lh}$ can serve as the uplifts in augmented profit function, defined by Galiana et al. (2003). Self-scheduling done by maximizing the augmented profit, a quantity defined by the profit without uplifts plus the uplift function, results in operational schedule. A generator therefore can no longer claim unequal treatment since the centralized schedule profit plus the uplift is equal to the self-schedule augmented profit at the existing price.

Computational performance The optimal profit functions $\omega^1$, $\omega^0$ are necessary to be determined in order to evaluate unit self-commitment schedule. The modelling of these variables seriously complicates the pricing model. The proper realization requires use of $2LH$ binary variables and formulating $8LH$ constraints.

During experimental analysis we noticed however, that the prices obtained in the model are not far from the marginal prices derived as dual prices to power balance constraint in the social welfare maximization model (Arroyo and Conejo, 2002). So we found very promising to use these marginal prices in our model to obtain a good initial feasible binary solution of problem (1)–(20). To do this we fix the selling prices $\pi^S_h$ to equal the marginal prices and solve the resulting restricted model (1)–(20). The model with fixed selling prices is easily solvable. It gives the upper bound of the compensations $R$ and an integer feasible solution. Our experience shows that the knowledge of such a feasible solution may improve drastically the computational effort.

Finally, let us conclude the analysis. Our complete market clearing price optimization model for social welfare distribution calculation is in the MILP form defined by constraints (2)–(20), with (1) as the minimized objective. Providing a good initial feasible integer solution is also highly recommended.

5. ILLUSTRATIVE EXAMPLE
The prices and other block quantities are unchanged, details are reported in Table 2. In hour 8 from 200 MWh to 100 MWh, to better illustrate the features of pricing model.

System load considered in (Li et al., 1997). We also reduced the volume of the second block quantities, which have been altered to preserve equality of total energy offered to buy and to be constant over market horizon are reported after Arroyo and Conejo (2002) in Table 1.

The technical data for all generation units are described in (Li et al., 1997). The sum of the coefficients of the exponential start-up cost function is taken as the start-up price, exactly the same as in (Arroyo and Conejo, 2002). Four-blocks selling energy price bids assumed to be constant over market horizon are reported after Arroyo and Conejo (2002) in Table 1.

In the example below we illustrate how the fair prices for the multiperiod auction can be found under newly proposed welfare distribution model. Realistic case study discussed in (Arroyo and Conejo, 2002) is served as the base for the data. The multiperiod pool-based auction clearing procedure is considered for a local pool-based energy market with 20 thermal units and one elastic, aggregated demand consumer (a retail company). The market horizon is spread over 24 consecutive hours. Each market participant is allowed to submit bids consisting of 4 energy-price blocks. The units sell offers may also contain the start-up bidding price and the technical constraints, which include the minimum and maximum operating limits as well as the minimum up time. In our example the ramp-rate limits are not considered.

The technical data for all generation units are described in (Li et al., 1997). The sum of the coefficients of the exponential start-up cost function is taken as the start-up price, exactly the same as in (Arroyo and Conejo, 2002). Four-blocks selling energy price bids assumed to be constant over market horizon are reported after Arroyo and Conejo (2002) in Table 1.

Bidding price and the technical constraints, which include the minimum and maximum operating limits as well as the minimum up time. In our example the ramp-rate limits are not considered.

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The technical data for all generation units are described in (Li et al., 1997). The sum of the coefficients of the exponential start-up cost function is taken as the start-up price, exactly the same as in (Arroyo and Conejo, 2002). Four-blocks selling energy price bids assumed to be constant over market horizon are reported after Arroyo and Conejo (2002) in Table 1.

Buying energy bids are also derived after Arroyo and Conejo (2002) except the first blocks quantities, which have been altered to preserve equality of total energy offered to buy and system load considered in (Li et al., 1997). We also reduced the volume of the second block in hour 8 from 200 MWh to 100 MWh, to better illustrate the features of pricing model. The prices and other blocks quantities are unchanged, details are reported in Table 2.

<table>
<thead>
<tr>
<th>Block</th>
<th>Units</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Q</td>
<td>P</td>
</tr>
<tr>
<td>B2</td>
<td>Q</td>
<td>P</td>
</tr>
<tr>
<td>B3</td>
<td>Q</td>
<td>P</td>
</tr>
<tr>
<td>B4</td>
<td>Q</td>
<td>P</td>
</tr>
</tbody>
</table>

Table 1: Structure of the selling energy bids: quantity 'Q' (MW) and price 'P' ($/MWh) defined in every block for every unit
Table 3 presents the results of the first step of market clearing procedure: determination of energy quantities to buy or sell from every bid. The maximum social welfare preserving all technical constraints is obtained, in total amount of 403 666.4$. It should be noticed, that although the solution preserves all constraints considered in (Arroyo and Conejo, 2002), it was found under capacity and minimum up time constraints only.

| L | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 |
| 2 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 | 130 |
| 3 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 | 460 |
| 5 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 6 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 |
| 7 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 | 455 |
| 8 | 470 | 470 | 443 | 403 | 458 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 | 470 |
| 9 | 40 | 40 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 | 162 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 | 55 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 | 2982 |

Table 3: Accepted production (L) and consumption (B) that maximize the social welfare

Let us look at the results shown in Table 3 in more detail. One can notice that in hours 8, 20 and 21 all committed units are running at full capacities, but this allows us to cover only part of offered blocks of buying energy bids. Thus fair consumer prices in those hours should be 25.5, 25.8 and 24.5 respectively. On the other hand minimum prices of unaccepted selling bids are 23.8 (minimum price of uncommitted unit 13). And the price of marginal working unit 12 is 24.32$/MWh.

Furthermore, the comparison of market clearing results with sell bids given in Table 1 shows that maximum accepted sell bids prices are greater than the minimum prices of rejected sell bids in hours 1–22. Obviously this poses significant difficulties in setting fair uniform selling prices, as there are generation units that may claim for not being accepted or for producing under non-competitive conditions. Thus the non-competitiveness conditions may appear both on the consumers and producers side.

To compute the market prices under proposed welfare distribution model firstly we have found a good initial feasible integer solution. This solution was found by fixing the selling prices to marginal prices m.p. shown in Table 4. Then the restricted model (1)–(20) was solved in less than 1 second. The solution evaluates the upper bound of the compensation costs to $\hat{R} = 246.04$.}

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Table 4: Selling $\pi_h^S$ and buying $\pi_h^B$ hourly prices minimizing compensation costs. Occurrence of a consumer or producer compensation is marked with a '*'.

Then, after communicating the initial solution to CPLEX, we have solved the model (1)–(20). Table 4 presents the market prices found under newly proposed welfare distribution model, with comparison to the marginal prices. The minimum compensations costs are $R = 228.22$. It should be noted that the compensation costs are almost unnoticeable in comparison to over 400,000 of the total social welfare. They ensure that all units and consumer operate in the best way, making optimal profit at the market prices.

The restricted and complete price clearing models (1)–(20) were both solved under 20 seconds, using CPLEX 9.1 on a Pentium D PC with 3GHz processor and 1GB of RAM.

In Figure 8 we show that the buying prices follow the shape of the consumption curve very closely. Generally, the buying and selling prices may be different in each hour, but it should be noticed that these prices differ only in hours 8 and 21 (marked gray). This difference allows Operator to cover the compensations paid to competitive participants whose offers are not or are only partially accepted, if their bidding prices are competitive comparing to the clearing market prices. The prices are marked with a '*' when hourly compensations occur.

In our example the consumer is compensated in the 2’nd and 8’th hour. In the 2’nd hour
consumer buys 2 MWh more than offered under the price equal to 23.11 $/MWh. On the other hand, in hour 8 consumption had to be reduced from offered 3335 MWh to 3322 MWh, due to the units constraints. In the other hours with high demand (20 and 21) buying prices are equal to consumer offered prices, thus competitiveness conditions are accomplished for consumer. The compensations paid to the consumers may be seen as rather a symbolic one, as they are less than 2 dollars \((23.11 - 23) \times 2 + (25.5 - 25.41) \times 13\).

From the consumer’s perspective it is easy to observe why the buying prices \(\pi_B\) are preserving the fairness conditions. For producers, however, there are no a simple general way of checking whether a single selling price \(\pi_S^h\) is fair or not, since all prices \(\pi_S\) support the production schedule in the whole horizon, due to the multiperiod constraints. It can be noticed however, that the cheapest and the most competitive units 1, 2, 3, 4, 6, 7, 14 and 16, that are running at full capacities during the whole horizon, are fairly priced as the selling prices are never less than 22$/MWh (the highest price of 16th unit is 21.92$/MWh). Thus the computed selling prices support their schedule. The same situation occurs with unit 8 and 9, which in most hours are working at maximum when the selling price is grater than their maximum offered price. The cheaper 8th unit always sells energy from the last block and the selling price is always equal to the last block price (22 $/MWh) if the unit is not running at full capacity (hours 3,4,5,13). We can observe, that in those hours the 9th unit sells energy below its offered price, but the production is equal to its minimum limits, thus the competitiveness conditions are also accomplished for this unit. On the other hand, most expensive uncommitted units 11, 15, 17, 18, 19, 20 have no incentives to turn on as the selling prices are never grater than their offered prices (26.37$/MWh at least).

Lets now focus on more expensive working units — unit 5 and 12. Detailed analysis shows that unit 5 could obtain grater profit producing more in hours 10 and 18 – and it gets compensations for lost opportunity of gaining surplus equal to 0.2$. The same happens with 12 unit – it receives 0.84$ in the 9th hour. In other hours the selling prices are always greater or equal to their offer prices if the production level exceeds their minimum production level. In hours when they run at minimum capacity they accept lower selling prices as staying online is profitable for them. In the last hours units 5 and 12 have no longer economic motivations to stay online, exactly as it was expected in the pool operational schedule.

Finally, we should turn our attention to the uncommitted units 10 and 13 that may be perhaps seen as competitive in some hours. In Figure 9 we show their possible maximal hourly profits if they would sell energy at market prices.

A detailed analysis of the units opportunity of making profit under computed selling prices shows however, that unit 10 couldn’t break even even if it would produce in any schedule. So the unit 10 has no incentives to change the Operator schedule.
Figure 9: Maximum possible hourly profit of selling energy at market prices for (a) unit 10, and (b) unit 13.

\[
\begin{array}{c|cc}
\text{Hours } h & \text{20} & \text{21} \\
\text{\(R_{13,h}^i\)} [\$] & 170.08 & 55.68 \\
\end{array}
\]

Table 5: Hourly commitment compensations paid to unit 13

The only unit that receives unit commitment compensations is unit 13. This unit stays uncommitted during the whole market horizon, however it could make some profit if it would produce in hours 1–21 at the market selling prices. The Operator assigns commitment compensations equal to 225.76\$. This amount is split among those hours where unit’s self schedule strays from the market determined schedule, according to model (21)-(25). It keeps the unit from changing the scheduling state in such way, that the market schedule becomes as good as the best option the unit can operate. Table 5 presents detailed description of the hourly commitment compensations assign to 13 unit.

6. CONCLUSIONS
A new pricing scheme presented in this paper for social welfare distribution in the multi-period auctions is as close as possible to a competitive electricity market clearing, i.e. the generating units are paid uniform hourly selling energy prices and consumers are paid uniform hourly buying prices. Differences in buying and selling prices that enable to cover the costs of necessary compensations paid to unfairly priced participants are calculated by a relatively simple mixed linear programming model that minimizes the compensation costs. The results of the example of a realistic case study reported in section 5 illustrate that the model provides fair schedules that allow every thermal unit and every consumer to obtain individual maximum profits. We believe that our model gives some incentives to generators to bid fairly, so the incongruence between the minimum-cost centralized schedule and the self-schedules is somehow reduced.

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REFERENCES


